Hyperbolic derivatives

Here is a chart of the six hyperbolic functions and their derivatives. Notice that the relationships between these functions and their derivatives is similar to those of regular trigonometric derivatives.

The only difference is that the derivative of $y = \cosh[g(x)]$ isn't $y' = -\sinh[g(x)][g'(x)]$, as you'd expect based on the regular trigonometric derivatives. Instead, with the hyperbolic function $y = \cosh[g(x)]$, we drop the negative sign when we take the derivative and therefore its derivative is $y' = \sinh[g(x)][g'(x)]$.

$y = \sinh[g(x)]$	$y' = \cosh[g(x)][g'(x)]$
$y = \cosh[g(x)]$	$y' = \sinh[g(x)][g'(x)]$
$y = \tanh[g(x)]$	$y' = \operatorname{sech}^2[g(x)][g'(x)]$
$y = \coth[g(x)]$	$y' = -\operatorname{csch}^2[g(x)][g'(x)]$
$y = \operatorname{sech}[g(x)]$	$y' = -\operatorname{sech}[g(x)]\tanh[g(x)][g'(x)]$
$y = \operatorname{csch}[g(x)]$	$y' = -\operatorname{csch}[g(x)]\coth[g(x)][g'(x)]$

Notice that the chart above also reminds you to apply chain rule, and multiply the derivative of the hyperbolic trigonometric function by the derivative of the inside function, which is $g'(x)$.

Let's try an example of an equation with a hyperbolic function.

Example

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Find the derivative.

$$y = 9\tanh\left(x^5\right)$$

Let's start by taking the derivative of the hyperbolic function (tanh, in this case), and then we'll apply chain rule and multiply by the derivative of the inside function (x^5, in this case).

$$y' = 9\text{sech}^2\left(x^5\right)\left(5x^4\right)$$

$$y' = 45x^4\text{sech}^2\left(x^5\right)$$

Let's try another example where the hyperbolic function occurs as part of a larger function.

Example

Find the derivative.

$$y = 2\sinh\left(4x^6\right) - 26x^2$$

First, take the derivative of the initial term $2\sinh\left(4x^6\right)$, and then move on to the second term, $-26x^2$.

$$y' = 2\cosh\left(4x^6\right)\left(24x^5\right) - 52x$$

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$$y' = 48x^5 \cosh\left(4x^6\right) - 52x$$

$$y' = 4x\left[12x^4 \cosh\left(4x^6\right) - 13\right]$$

Let's try another more complex example.

Example

Find the derivative.

$$y = 8x^{-4} - \operatorname{csch}\left(3x^2\right) + \frac{\sinh\left(x^3\right)}{\tanh\left(2x^7\right)}$$

We'll take the derivative term by term, using quotient rule to find the derivative of the last term.

$$y' = -32x^{-5} - \left[-\operatorname{csch}\left(3x^2\right)\coth\left(3x^2\right)(6x)\right] + \frac{\cosh\left(x^3\right)\left(3x^2\right)\tanh\left(2x^7\right) - \sinh\left(x^3\right)\operatorname{sech}^2\left(2x^7\right)\left(14x^6\right)}{\left[\tanh\left(2x^7\right)\right]^2}$$

$$y = -\frac{32}{x^5} + 6x\operatorname{csch}\left(3x^2\right)\coth\left(3x^2\right) + \frac{3x^2\cosh\left(x^3\right)\tanh\left(2x^7\right) - 14x^6\sinh\left(x^3\right)\operatorname{sech}^2\left(2x^7\right)}{\tanh^2\left(2x^7\right)}$$

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